Unit 05

FACTORIZATION

Factorization: If a polynomial p(x) can be expressed as p(x) = g(x) h (x), then each of the polynomials g(x) and h(x) is called a factor of p(x).
The process of finding the factors is called factorization.

(a) Factorization of the Expression of the type ka + kb + kc.

Example

Factorize 5a-5b+5c

Solution

$5a - 5b + 5c = 5(a - b + c)$

Example

Factorize $5a - 5b - 15c$

Solution:

$5a - 5b - 15c = 5(a - b - 3c)$

(b) Factorization of the Expression of the type ac + ad + bc + bd

We can write ac + ad + bc + bd as

$(ac + ad) + (bc + db)$

$= a(c + d) + b(c + d)$

$= (a + b)(c + d)$

Example

Factorize $3x - 3a + xy - ay$

Solution:

Regrouping the terms of given polynomial

$3x + xy - 3a - ay = x(3 + y) - a(3 + y)$

$= (3 + y)(x - a)$

Example

Factorize $pqr + qr^2 - pr^2 - r^3$

Solution:

The given expression $= r(pq + qr - pr - r^2)$

$= r\left[(pq + qr) - pr - r^2\right]$

$= r\left[q(p + r) - r(p + r)\right]$

$= r(p + r)(q - r)$

(c) Factorization of the Expression of the type $a^2 \pm 2ab + b^2$.

We know that

(i) $a^2 + 2ab + b^2 = (a+b)^2 = (a+b)(a+b)$

(ii) $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$

Example

Factorization $25x^2+16+40x$.

Solution:

$25x^2+40x+16 = (5x)^2+2(5x)(4) + (4)^2$

$= (5x+4)^2$

$= (5x+4)(5x+4)$

Example

Factorize $12x^2-36x+27$

Solution:

$12x^2 - 36x + 27 = 3(4x^2 - 12x + 9)$

$= 3\left[(2x)^2 - 2(2x)(3) + (3)^2\right]$

$= 3(2x - 3)^2$

$= 3(2x - 3)(2x - 3)$

(d) Factorization of the Expression of the type $a^2 - b^2$.

Example Factorize

(i) $4x^2-(2y-z)^2$ (ii) $6x^4-96$

Solution

(i) $4x^2-(2y-z)^2 = (2x)^2-(2y-z)^2$

$$= [2x-(2y-z)][2x+(2y-z)]$$

$$= (2x-2y+z)(2x+2y-z)$$

(ii) $6x^4-96 = 6(x^4-16)$

$$= 6\left[(x^2)^2-(4)^2\right]$$

$$= 6(x^2-4)(x^2+4)$$

$$= 6\left[(x)^2-(2)^2\right](x^2+4)$$

$$= 6(x-2)(x+2)(x^2+4)$$

(e) Factorization of the Expression of the types $a^2 \pm 2ab + b^2 - c^2$.

We know that

$$a^2 \pm 2ab + b^2 - c^2 = (a \pm b)^2 - (c)^2 = (a \pm b - c)(a \pm b + c)$$

Example

Factorize (i) $x^2+6x+9-4y^2$

(ii) $1+2ab-a^2-b^2$

Solution:

(i) $x^2+6x+9-4y^2 = (x+3)^2-(2y)^2$

$$= (x+3+2y)(x+3-2y)$$

(ii) $1+2ab-a^2-b^2 = 1-(a^2-2ab+b^2)$

$$= (1)^2-(a-b)^2$$

$$= [1-(a-b)][1+(a-b)]$$

$$= (1-a+b)(1+a-b)$$

Exercise 5.1

Q.1 Factorize

(i) $2abc-4abx+2abd$

$= 2ab(c-2x+d)$

(ii) $9xy-12x^2y+18y^2$

$= 3y(3x-4x^2+6y)$

(iii) $-3x^2y-3x+9xy^2$

$= -3x(xy+1-3y^2)$

(iv) $5ab^2c^3-10a^2b^3c+20a^3bc^2$

$=5abc(bc^2-2ab^2+4a^2c)$

(v) $3x^3y(x-3y)-7x^2y^2(x-3y)$

$(x-3y)(3x^3y-7x^2y^2)$

$(x-3y).x^2y(3x-7y)$

$\Rightarrow$ $x^2y(x-3y)(3x-7y)$

(vi) $2xy^3(x^2+5)+8xy^2(x^2+5)$

$(x^2+5)(2xy^3+8xy^2)$

$(x^2+5)\ 2xy^2(y+4)$

$=2xy^2(x^2+5)(y+4)$

Q.2 (i) $5ax-3ay-5bx+3by$

$=5ax-5bx-3ay+3by$

$=5x(a-b)-3y(a-b)$

$=(a-b)(5x-3y)$

(ii) $3xy+2y-12x-8$

$=3xy-12x+2y-8$

$=3x(y-4)+2(y-4)$

$=(y-4)(3x+2)$

(iii) $x^3+3xy^2-2x^2y-6y^3$

$=x^3-2x^2y+3xy^2-6y^3$

$=x^2(x-2y)+3y^2(x-2y)$

$=(x-2y)(x^2+3y^2)$

(iv) $(x^2-y^2)z+(y^2-z^2)x$

$=x^2z-y^2z+y^2x-z^2x$

$=x^2z-z^2x+y^2x-y^2z$

$=xz(x-z)+y^2(x-z)$

$=(x-z)(xz+y^2)$

Q.3 (i) $144a^2+24a+1$

$=(12a)^2+2(12a)(1)+(1)^2$

$=(12a+1)^2$

$=(12a+1)(12a+1)$

(ii) $\dfrac{a^2}{b^2}-2+\dfrac{b^2}{a^2}$

$=\left(\dfrac{a}{b}\right)^2-2\left(\dfrac{a}{b}\right)\left(\dfrac{b}{a}\right)+\left(\dfrac{b}{a}\right)^2$

$=\left(\dfrac{a}{b}-\dfrac{b}{a}\right)^2$

$=\left(\dfrac{a}{b}-\dfrac{b}{a}\right)\left(\dfrac{a}{b}-\dfrac{b}{a}\right)$

(iii) $(x+y)^2-14z(x+y)+49z^2$

$=(x+y)^2-2(x+y)(7z)+(7z)^2$

$=(x+y-7z)^2$

$=(x+y-7z)(x+y-7z)$

(iv) $12x^2-36x+27$

$=3(4x^2-12x+9)$

$=3\left[(2x)^2-2(2x)(3)+(3)^2\right]$

$=3(2x-3)^2$

$=3(2x-3)(2x-3)$

Q.4 (i) $3x^2-75y^2$

$=3(x^2-25y^2)$

$=3\left[(x)^2-(5y)^2\right]$

$=3(x+5y)(x-5y)$

(ii) $x(x-1)-y(y-1)$

$=x^2-x-y^2+y$

$=x^2-y^2-x+y$

$=(x+y)(x-y)-1(x-y)$

$=(x-y)(x+y-1)$

(iii) $128am^2-242an^2$

$=2a(64m^2-121n^2)$

$=2a\left[(8m)^2-(11n)^2\right]$

$=2a(8m+11n)(8m-11n)$

(iv) $3x-243x^3$

$=3x(1-81x^2)$

$=3x\left[(1)^2-(9x)^2\right]$

$=3x(1+9x)(1-9x)$

Q.5 (i) x^2-y^2-6y-9

$=x^2-(y^2+6y+9)$

$=x^2-\left[(y)^2+2(y)(3)+(3)^2\right]$

$=(x)^2-(y+3)^2$

$=[(x)+(y+3)][(x)-(y+3)]$

$=(x+y+3)(x-y-3)$

(ii) x^2-a^2+2a-1

$=x^2-(a^2-2a+1)$

$=(x)^2-(a-1)^2$

$=[(x)+(a-1)][(x)-(a-1)]$

$=(x+a-1)(x-a+1)$

(iii) $4x^2-y^2-2y-1$

$=4x^2-(y^2+2y+1)$

$=(2x)^2-(y+1)^2$

$=[(2x)+(y+1)][(2x-(y+1)]$

$=(2x+y+1)(2x-y-1)$

(iv) $x^2-y^2-4x-2y+3$

$=x^2-y^2-4x-2y+4-1$

$=x^2-4x+4-y^2-2y-1$

$=(x)^2-2(x)(2)+(2)^2-(y^2+2y+1)$

$=(x-2)^2-(y+1)^2$

$=[(x-2)+(y+1)][(x-2)-(y+1)]$

$=(x-2+y+1)(x-2-y-1)$

$=(x+y-1)(x-y-3)$

(v) $25x^2-10x+1-36z^2$

$=(5x)^2-2(5x)(1)+(1)^2-(6z)^2$

$=(5x-1)^2-(6z)^2$

$=[(5x-1)+(6z)][(5x-1)-(6z)]$

$=(5x-1+6z)(5x-1-6z)$

$=(5x+6z-1)(5x-6z-1)$

(vi) $x^2-y^2-4xz+4z^2$

$=x^2-4xz+4z^2-y^2$

$=(x)^2-2(x)(2z)+(2z)^2-(y)^2$

$=(x-2z)^2-(y)^2$

$=[(x-2z)+(y)][(x-2z)-(y)]$

$=(x-2z+y)(x-2z-y)$

(a) Factorization of the Expression of types $a^4+a^2b^2+b^4$ or a^4+4b^4

Factorization of such types of expression is explained in the following examples.

Example

Factorize $81x^4+36x^2y^2+16y^4$

Solution

$81x^4+36x^2y^2+16y^4$

$=(9x^2)^2+72x^2y^2+(4y^2)^2-36x^2y^2$

$=(9x^2)^2+(4y^2)^2+2(9x^2)(4y^2)-36x^2y^2$

$$= (9x^2 + 4y^2)^2 - (6xy)^2$$
$$= (9x^2 + 4y^2 + 6xy)(9x^2 + 4y^2 - 6xy)$$
$$= (9x^2 + 6xy + 4y^2)(9x^2 - 6xy + 4y^2)$$

Example

Factorize $9x^4 + 36y^4$

Solution:

$9x^4 + 36y^4$

$$= 9x^4 + 36y^4 + 36x^2y^2 - 36x^2y^2$$
$$= (3x^2)^2 + 2(3x^2)(6y^2) + (6y^2)^2 - (6xy)^2$$
$$= (3x^2 + 6y^2)^2 - (6xy)^2$$
$$= (3x^2 + 6y^2 + 6xy)(3x^2 + 6y^2 - 6xy)$$
$$= (3x^2 + 6xy + 6y^2)(3x^2 - 6xy + 6y^2)$$

(b) Factorization of the Expression of the type $x^2 + px + q$.

Example

Factorize (i) $x^2 - 7x + 12$

(ii) $x^2 + 15x - 36$

Solution:

(i) $x^2 - 7x + 12$

From the factors of 12 the suitable pair of numbers is -3 and -4 since

$(-3) + (-4) = -7$ *and* $(-3)(-4) = 12$

Hence $x^2 - 7x + 12 = x^2 - 3x - 4x + 12$

$= x(x-3) - 4(x-3)$

$= (x-3)(x-4)$

(ii) $x^2 + 5x - 36$

From the possible factors of 36, the suitable pair is 9 and -4 because $9 + (-4) = 5$ and $9 \times (-4) = -36$

Hence $x^2 + 5x - 36 = x^2 + 9x - 4x - 36$

$= x(x+9) - 4(x+9)$

$= (x+9)(x-4)$

(c) Factorization of the Expression of the type $ax^2 + bx + c$, $a \neq 0$

Example

Factorize (i) $9x^2 + 21x - 8$

(ii) $2x^2 - 8x - 42$

(iii) $10x^2 - 41xy + 21y^2$

Solution:

(i) $9x^2 + 21x - 8$

In this case, on comparing with $ax^2 + bx + c$, $ac = (9)(-8) = -72$.

From the possible factors of 72 the suitable pair of numbers (with proper sign) is 24 and -3 whose Sum $= 24 + (-3) = 21$, (the coefficient of x)

And their product $= (24)(-3) = -72 = \text{ac}$

Hence $9x^2 + 21x - 8$

$$= 9x^2 + 24x - 3x - 8$$
$$= 3x(3x+8) - 1(3x+8)$$
$$= (3x+8)(3x-1)$$

(ii) $2x^2 - 8x - 42 = 2(x^2 - 4x - 21)$

Comparing

$x^2 - 4x - 21$ *with* $ax^2 + bx + c$

We have ac $= (+1)(-21) = -21$

From the possible factors of 21 the suitable pair of numbers is -7 and $+3$ whose

Sum $= -7 + 3 = -4$ and product $= (-7)(3) = -21$

Hence $x^2 - 4x - 21$

$$= x^2 + 3x - 7x - 21$$
$$= x(x+3) - 7(x+3)$$

$=(x+3)(x-7)$

Hence $2x^2-8x-42=2(x^2-4x-21)$
$=2(x+3)(x-7)$

(iii) $10x^2-41xy+21y^2$

Here $ac=(10)(21)=210$

Two suitable factors of 210 are −35 and −6.

Their sum = −35 −6 = -41

And product = (−35) (−6) = 210

Hence $10x^2-41xy+21y^2$

$$=10x^2-35xy-6xy+21y^2$$
$$=5x(2x-7y)-3y(2x-7y)$$
$$=(2x-7y)(5x-3y)$$

(d) Factorization of the following types of Expressions.

$(ax^2+b+c)(ax^2+bx+d)+k$

$(x+a)(x+b)(x+c)(x+d)+k$

$(x+a)(x+b)(x+c)(x+d)+kx^2$

Example

Factorize $(x^2-4x-5)(x^2-4x-12)-144$

Solution:

$(x^2-4x-5)(x^2-4x-12)-144$

Let $y=x^2-4x$. Then

$$(y-5)(y-12)-144 = y^2-17y+60-144$$
$$=y^2-17y-84$$
$$=y^2-21y+4y-84$$
$$=y(y-21)+4(y-21)$$
$$=(y-21)(y+4)$$
$$=(x^2-4x-21)(x^2-4x+4) \quad (\textit{Since } y=x^2-4x)$$
$$=(x^2-7x+3x-21)\left[(x)^2-2(x)(2)+(2)^2\right]$$
$$=\left[x(x-7)+3(x-7)\right](x-2)^2$$
$$=(x-7)(x+3)(x-2)(x-2)$$

Example

Factorize

$(x+1)(x+2)(x+3)(x+4)-120$

Solution:

We observe that $1+4=2+3$.

It suggests that we rewrite the given expression as

$[(x+1)(x+4)][(x+2)(x+3)]-120$

$(x^2+5x+4)(x^2+5x+6)-120$

Let $x^2+5x=y$, then

We get $(y+4)(y+6)-120$

$=y^2+10y+24-120$

$=y^2+10y-96$

$=y^2+16y-6y-96$

$=y(y+16)-6(y+16)$

$=(y+16)(y-6)$

$=(x^2+5x+16)(x^2+5x-6)$ (since $y=x^2+5x$)

$=(x^2+5x+16)\left[x^2+6x-x-6\right]$

$=(x^2+5x+16)[(x+6)-1(x+6)]$

$=(x^2+5x+16)(x+6)(x-1)$

Example

Factorize $(x^2-5x+6)(x^2+5x+6)-2x^2$

Solution:

$(x^2-5x+6)(x^2+5x+6)-2x^2$

$=\left[x^2-3x-2x+6\right]\left[x^2+3x+2x+6\right]-2x^2$

$=[x(x-3)-2(x-3)]\,[x(x+3)+2(x+3)]-2x^2$

$=[(x-3)(x-2)][(x+3)(x+2]-2x^2$

$=[(x-2)(x+2)][(x-3)(x+3)]-2x^2$

$=(x^2-4)(x^2-9)-2x^2$

$=x^4-13x^2+36-2x^2$

$=x^4-15x^2+36$

$=x^4-12x^2-3x^2+36$

$=x^2(x^2-12)-3(x^2-12)$

$=(x^2-12)(x^2-3)$

$=\left[(x)^2-(2\sqrt{3})^2\right]\left[(x)^2-(\sqrt{3})^2\right]$

$=(x-2\sqrt{3})(x+2\sqrt{3})(x-\sqrt{3})(x+\sqrt{3})$

(c) Factorization of Expressions of the following Types

$a^3+3a^2b+3ab^2+b^3$

$a^3-3a^2b+3ab^2-b^3$

Example:

Factorize $x^3-8y^3-6x^2y+12xy^2$

Solution:

$x^3-8y^3-6x^2y+12xy^2$

$=(x)^3-(2y)^3-3(x)^2(2y)+3(x)(2y)^2$

$=(x)^3-3(x)^2(2y)+3(x)(2y)^2-(2y)^3$

$=(x-2y)^3$

$=(x-2y)(x-2y)(x-2y)$

(d) Factorization of Expressions of the following types $a^3\pm b^3$

We recall the formulas,

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

Example

Factorize $27x^3+64y^3$

Solution:

$27x^3+64y^3 \quad =(3x)^3+(4y)^3$

$=(3x+4y)\left[(3x)^2-(3x)(4y)+(4y)^2\right]$

$=(3x+4y)(9x^2-12xy+16y^2)$

Example

Factorize $1-125x^3$

Solution

$$1-25x^3 = (1)^3-(5x)^3$$

$$=(1-5x)\left[(1)^2+(1)(5x)+(5x)^2\right]$$

$$=(1-5x)(1+5x+25x^2)$$

Exercise 5.2

Q.1 Factorize

(i) $x^4+\frac{1}{x^4}-3$

$$=x^4+\frac{1}{x^4}-2-1$$

$$=(x^2)^2+\left(\frac{1}{x^2}\right)^2-2(x^2)\left(\frac{1}{x^2}\right)-1$$

$$=\left(x^2-\frac{1}{x^2}\right)^2-(1)^2$$

$$=\left(x^2-\frac{1}{x^2}+1\right)\left(x^2-\frac{1}{x^2}-1\right)$$

(ii) $3x^4+12y^4$

$$=3(x^4+4y^4)$$

$$=3\left[(x^2)^2+(2y^2)^2+2(x^2)(2y^2)-4x^2y^2\right]$$

$$=3\left[(x^2+2y^2)^2-(2xy)^2\right]$$

$$=3(x^2+2y^2+2xy)(x^2+2y^2-2xy)$$

$$=3(x^2+2xy+2y^2)(x^2-2xy+2y^2)$$

(iii) $a^4+3a^2b^2+4b^4$

$$a^4+4a^2b^2+4b^4-a^2b^2$$

$$=(a^2)^2+2(a^2)(2b^2)+(2b^2)^2-a^2b^2$$

$$=(a^2+2b^2)^2-(ab)^2$$

$$=(a^2+2b^2+ab)(a^2+2b^2-ab)$$

$$=(a^2+ab+2b^2)(a^2-ab+2b^2)$$

(iv) $4x^4+81$

$$=(2x^2)^2+(9)^2+2(2x^2)(9)-36x^2$$

$$=(2x^2+9)^2-(6x)^2$$

$$=(2x^2+9+6x)(2x^2+9-6x)$$

$$=(2x^2+6x+9)(2x^2-6x+9)$$

(v) x^4+x^2+25

$$=(x^2)^2+2(x^2)(5)+(5)^2-9x^2$$

$$=(x^2+5)^2-(3x)^2$$

$$=(x^2+5+3x)(x^2+5-3x)$$

$$=(x^2+3x+5)(x^2-3x+5)$$

(vi) x^4+4x^2+16

$$=(x^2)^2+2(x^2)(4)+(4)^2-4x^2$$

$$=(x^2+4)^2-(2x)^2$$

$$=(x^2+4+2x)(x^2+4-2x)$$

$$=(x^2+2x+4)(x^2-2x+4)$$

Q.2 (i) $x^2+14x+48$

$$=x^2+6x+8x+48$$

$$=x(x+6)+8(x+6)$$

$$=(x+6)(x+8)$$

(ii) $x^2-21x+108$

$$=x^2-9x-12x+108$$

$$=x(x-9)-12(x-9)$$

$=(x-9)(x-12)$

(iii) $x^2-11x-42$

$=x^2+3x-14x-42$

$=x(x+3)-14(x+3)$

$=(x+3)(x-14)$

(iv) $x^2+x-132$

$=x^2+12x-11x-132$

$=x(x+12)-11(x+12)$

$=(x+12)(x-11)$

Q.3 (i) $4x^2+12x+5$

$=4x^2+2x+10x+5$

$=2x(2x+1)+5(2x+1)$

$=(2x+1)(2x+5)$

(ii) $30x^2+7x-15$

$=30x^2+25x-18x-15$

$=5x(6x+5)-3(6x+5)$

$=(6x+5)(5x-3)$

(iii) $24x^2-65x+21$

$=24x^2-56x-9x+21$

$=8x(3x-7)-3(3x-7)$

$=(3x-7)(8x-3)$

(iv) $5x^2-16x-21$

$=5x^2+5x-21x-21$

$=5x(x+1)-21(x+1)$

$=(x+1)(5x-21)$

(v) $4x^2-17xy+4y^2$

$=4x^2-16xy-xy+4y^2$

$=4x(x-4y)-y(x-4y)$

$=(x-4y)(4x-y)$

(vi) $3x^2-38xy-13y^2$

$=3x^2-39xy+xy-13y^2$

$=3x(x-13y)+y(x-13y)$

$=(x-13y)(3x+y)$

(vii) $5x^2+33xy-14y^2$

$=5x^2+35xy-2xy-14y^2$

$=5x(x+7y)-2y(x+7y)$

$=(x+7y)(5x-2y)$

(viii) $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$

$=\left(5x-\frac{1}{x}\right)^2+2\left(5x-\frac{1}{x}\right)(2)+(2)^2$

$=\left(5x-\frac{1}{x}+2\right)^2$

$=\left(5x-\frac{1}{x}+2\right)\left(5x-\frac{1}{x}+2\right)$

Q.4 (i) $(x^2+5x+4)(x^2+5x+6)-3$

Let $x^2+5x=y$

then

$(x^2+5x+4)(x^2+5x+6)-3$

$=(y+4)(y+6)-3$

$=y^2+4y+6y+24-3$

$=y^2+10y+21$

$=y^2+3y+7y+21$

$=y(y+3)+7(y+3)$

$=(y+3)(y+7)$

Putting value of y

$=(x^2+5x+3)(x^2+5x+7)$

(ii) $(x^2-4x)(x^2-4x-1)-20$

Let $\quad x^2-4x=y$

then

$$(x^2-4x)(x^2-4x-1)-20$$
$$=y(y-1)-20$$
$$=y^2-y-20$$
$$=y^2+4y-5y-20$$
$$=y(y+4)-5(y+4)$$
$$=(y+4)(y-5)$$

Putting value of y

$$=(x^2-4x+4)(x^2-4x-5)$$
$$=\left[(x)^2-2(x)(2)+(2)^2\right]\left[x^2+x-5x-5\right]$$
$$=(x-2)^2\left[x(x+1)-5(x+1)\right]$$
$$=(x-2)^2(x+1)(x-5)$$

(iii) $\quad (x+2)(x+3)(x+4)(x+5)-15$

$$=\left[(x+2)(x+5)\right]\left[(x+3)(x+4)\right]-15$$
$$=(x^2+2x+5x+10)(x^2+3x+4x+12)-15$$
$$=(x^2+7x+10)(x^2+7x+12)-15$$

Let $\quad x^2+7x=y$

$$=(y+10)(y+12)-15$$
$$=y^2+10y+12y+120-15$$
$$=y^2+22y+105$$
$$=y^2+7y+15y+105$$
$$=y(y+7)+15(y+7)$$
$$=(y+7)(y+15)$$

Putting value of 'y'

$$(x^2+7x+7)(x^2+7x+15)$$

(iv) $\quad (x+4)(x-5)(x+6)(x-7)-504$

$$=(x^2+4x-5x-20)(x^2+6x-7x-42)-504$$
$$=(x^2-x-20)(x^2-x-42)-504$$

Let $\quad x^2-x=y$

$$=(y-20)(y-42)-504$$
$$=y^2-20y-42y+840-504$$
$$=y^2-62y+336$$
$$=y^2-6y-56y+336$$
$$=y(y-6)-56(y-6)$$
$$=(y-6)(y-56)$$

Putting value of 'y'

$$=(x^2-x-6)(x^2-x-56)$$
$$=(x^2+2x-3x-6)(x^2+7x-8x-56)$$
$$=\left[x(x+2)-3(x+2)\right]\left[x(x+7)-8(x+7)\right]$$
$$=(x+2)(x-3)(x+7)(x-8)$$

(v) $\quad (x+1)(x+2)(x+3)(x+6)-3x^2$

$$=(x+1)(x+6)(x+2)(x+3)-3x^2$$
$$=(x^2+x+6x+6)(x^2+2x+3x+6)-3x^2$$
$$=(x^2+6+7x)(x^2+6+5x)-3x^2$$
$$=\frac{x^2}{x^2}\left[(x^2+6+7x)(x^2+6+5x)-3x^2\right]$$
$$=x^2\left[\frac{(x^2+6+7x)(x^2+6+5x)}{x^2}-\frac{3x^2}{x^2}\right]$$
$$=x^2\left[\left(x+\frac{6}{x}+7\right)\left(x+\frac{6}{x}+5\right)-3\right]$$

Let $\quad x+\dfrac{6}{x}=y$

$$=x^2\left[(y+7)(y+5)-3\right]$$
$$=x^2(y^2+7y+5y+35-3)$$
$$=x^2(y^2+12y+32)$$
$$=x^2(y^2+4y+8y+32)$$
$$=x^2\left[y(y+4)+8(y+4)\right]$$
$$=x^2(y+4)(y+8)$$

Putting value of y

$$=x^2\left(x+\frac{6}{x}+4\right)\left(x+\frac{6}{x}+8\right)$$

$$=x^2\left(\frac{x^2+4x+6}{x}\right)\left(\frac{x^2+8x+6}{x}\right)$$

$$=(x^2+4x+6)(x^2+8x+6)$$

$$=(x^2+4x+6)(x^2+8x+6)$$

Q.5

(i) $x^3+48x-12x^2-64$

$$=x^3-12x^2+48x-64$$

$$=(x)^3-3(x^2)(4)+3(x)(4)^2-(4)^3$$

$$=(x-4)^3$$

$$=(x-4)(x-4)(x-4)$$

(ii) $8x^3+60x^2+150x+125$

$$=(2x)^3+3(2x)^2(5)+3(2x)(5)^2+(5)^3$$

$$=(2x+5)^3$$

$$=(2x+5)(2x+5)(2x+5)$$

(iii) $x^3-18x^2+108x-216$

$$=(x)^3-3(x)^2(6)+3(x)(6)^2-(6)^3$$

$$=(x-6)^3$$

$$=(x-6)(x-6)(x-6)$$

(iv) $8x^3-125y^3-60x^2y+150xy^2$

$$=8x^3-60x^2y+150xy^2-125y^3$$

$$=(2x)^3-3(2x)^2(5y)+3(2x)(5y)^2-(5y)^3$$

$$=(2x-5y)^3$$

$$=(2x-5y)(2x-5y)(2x-5y)$$

Q.6 (i) $27+8x^3$

$$=(3)^3+(2x)^3$$

$$=(3+2x)\left[(3)^2-(3)(2x)+(2x)^2\right]$$

$$=(3+2x)(9-6x+4x^2)$$

or $=(2x+3)(4x^2-6x+9)$

(ii) $125x^3-216y^3$

$$=(5x)^3-(6y)^3$$

$$=(5x-6y)\left[(5x)^2+(5x)(6y)+(6y)^2\right]$$

$$=(5x-6y)(25x^2+30xy+36y^2)$$

(iii) $64x^3+27y^3$

$$=(4x)^3+(3y)^3$$

$$=(4x+3y)\left[(4x)^2-(4x)(3y)+(3y)^2\right]$$

$$=(4x+3y)(16x^2-12xy+9y^2)$$

(iv) $8x^3+125y^3$

$$=(2x)^3+(5y)^3$$

$$=(2x+5y)\left[(2x)^2-(2x)(5y)+(5y)^2\right]$$

$$=(2x+5y)(4x^2-10xy+25y^2)$$

Remainder Theorem

If a polynomial p(x) is divided by a linear divisor $(x-a)$, then the remainder is $p(a)$.

Proof

Let $q(x)$ be the quotient obtained after dividing $p(x)$ by $(x-a)$. But the divisor $(x-a)$ is linear. So the remainder must be of degree zero i.e., a non-zero constant, say R. Consequently, by division Algorithm we may write.

$$p(x)=(x-a)q(x)+R$$

This is an identity in x and so is true for all real numbers x. In particular, it is true for $x=a$. Therefore,

$$p(a)=(a-a)q(a)+R=0+R=R$$

i.e., $p(a)=$ the remainder.

Hence the theorem.

Note: Similarly, if the divisor is (ax–b), we have

$$p(x)=(ax-b)q(x)+R$$

Substituting $x=\frac{b}{a}$ so that ax − b = 0, we obtain

$$p\left(\frac{b}{a}\right)=0.\ q\left(\frac{b}{a}\right)+R=0+R=R$$

Thus if the divisor is linear, the above theorem provides an efficient way of finding the remainder without being involved in the process of long division.

To find remainder (without dividing) when a polynomial is divided by a Linear Polynomial

Example

Find the remainder when $9x^2-6x+2$ is divided by

(i) $x-3$ (ii) $x+3$

(iii) $3x+1$ (iv) x

Solution:

Let $p(x)=9x^2-6x+2$

(i) When $p(x)$ is divided by $x-3$, by Remainder Theorem, the remainder is:

$$R=p(3)=9(3)^2-6(3)+2=65$$
$$=9(9)-18+2$$
$$\text{P}(3)=81-16$$
$$=65$$

(ii) When $p(x)$ is divided by $x+3=x-(-3)$, the remainder is

$$R=p(-3)=9(-3)^2-6(-3)+2$$
$$=9(9)+18+2$$
$$=81+20=101$$

(iii) When $p(x)$ is divided by $3x+1$, the remainder is

$$R=p\left(-\frac{1}{3}\right)=9\left(-\frac{1}{3}\right)^2-6\left(-\frac{1}{3}\right)+2=5$$

(iv) When $p(x)$ is divided by x, the remainder is

$$R=p(0)=9(0)^2-6(0)+2=2$$

Example

Find the value of k is the expression x^3+kx^2+3x-4 leaves a remainder of -2 when divided by $x+2$.

Solution:

Let $p(x)=x^3+kx^2+3x-4$.

By the remainder Theorem, when $p(x)$ is divided by $x+2=x-(-2)$, the remainder is:

$$p(-2)=(-2)^3+k(-2)^2+3(-2)-4$$
$$=-8+4k-6-4$$
$$=4k-18$$

By the given condition, we have

$p(-2)=-2 \quad\Rightarrow\quad 4k-18=-2$

$\Rightarrow \quad k=4$

5.2.3 Zero of a polynomial

If a specific number $x=a$ is substituted for a variable x in a polynomial $p(x)$ so that the value $p(a)$ is zero, then $x=a$ is called a zero of the polynomial $p(x)$.

Factor Theorem

The polynomial $(x-a)$ is a factor of the polynomial $p(x)$ if and only if $p(a)=0$.

Proof:

Let $q(x)$ be the quotient and R the remainder when a polynomial $p(x)$ is divided by $(x-a)$. Then by division Algorithm,

$$p(x)=(x-a)q(x)+R$$

By the Remainder Theorem, R = p(a).

Hence $p(x) = (x-a)q(x) + p(a)$

(i) Now if $p(a) = 0$, then $p(x) = (x-a)q(x)$

i.e., $(x-a)$ is a factor of $p(x)$.

(ii) Conversely, if $(x-a)$ is a factor of $p(x)$, then the remainder upon dividing $p(x)$ by $(x-a)$ must be zero i.e., p(a) = 0.

Example

Determine if $(x-2)$ is a factor of $x^3 - 4x^2 + 3x + 2$.

Solution:

Let

$$p(x) = x^3 - 4x^2 + 3x + 2$$

Then the remainder for $(x-2)$ is:

$$p(2) = (2)^3 - 4(2)^2 + 3(2) + 2$$
$$= 8 - 16 + 6 + 2 = 0$$

Hence by Factor Theorem, $(x-2)$ is a factor of the polynomial $p(x)$.

Example

Find a polynomial $p(x)$ of degree 3 that has 2, −1, and 3 as zeros (i.e., roots).

Solution:

Since $x = 2, -1, 3$ are roots of $p(x) = 0$.

So by Factor theorem $(x-2), (x+1)$ *and* $(x-3)$ are the factors of $p(x)$.

Thus $p(x) = a(x-2)(x+1)(x-3)$

Where any non-zero value can be assigned to a.

Taking a = 1, we get

$$p(x) = (x-2)(x+1)(x-3)$$
$$= x^3 - 4x^2 + x + 6$$ as the required polynomial.

Exercise 5.3

Q.1 Use the remainder theorem to find the remainder, when.

(i) $3x^3 - 10x^2 + 13x - 6$ is divided by $(x-2)$

Sol:

Let $P(x) = 3x^3 - 10x^2 + 13x - 6$

When P(x) is divided by x − 2 by remainder theorem, the remainder is:

$$R = P(2) = 3(2)^3 - 10(2)^2 + 13(2) - 6$$
$$= 3(8) - 10(4) + 26 - 6$$
$$= 24 - 40 + 26 - 6$$
$$= 50 - 46$$
$$= 4$$

(ii) $4x^3 - 4x + 3$ is divided by $(2x-1)$

Sol:

Let $P(x) = 4x^3 - 4x + 3$ when P(x) is divided by 2x − 1 by remainder theorem, the remainder is

$$R = P\left(\frac{1}{2}\right) \qquad = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3$$
$$= 4\left(\frac{1}{8}\right) - 2 + 3$$
$$= \frac{1}{2} + 1$$
$$= \frac{1+2}{2}$$

$R \quad = \frac{3}{2}$

(iii) $6x^4+2x^3-x+2$ is divided by $(x+2)$

Sol:

Let P (x) = $6x^4+2x^3-x+2$ when P(x) is divided by x + 2 by remainder theorem, the remainder is

$$R = P(-2) = 6(-2)^4+2(-2)^3-(-2)+2$$
$$= 6(16)+2(-8)+2+2$$
$$= 96-16+4$$
$$= 80+4$$
$$R = 84$$

(iv) $(2x-1)^3+6(3+4x)^2-10$ is divided by 2x + 1

Sol:

Let p(x) = $(2x-1)^3+6(3+4x)^2-10$ when P(x) is divided by 2x + 1 by remainder theorem, then remainder is

$$R = p\left(-\frac{1}{2}\right) = \left[2\left(-\frac{1}{2}\right)-1\right]^3+6\left[3+4\left(-\frac{1}{2}\right)\right]^2-10$$
$$= (-1-1)^3+6(3-2)^2-10$$
$$= (-2)^3+6(1)^2-10$$
$$= -8+6-10$$
$$= -12$$

(v) $x^3-3x^2+4x-14$ is divided by x + 2

Sol:

Let P(x) = $x^3-3x^2+4x-14$ when P(x) is divided by x + 2 by remainder theorem, then remainder is

$$R = P(-2) = (-2)^3-3(-2)^2+4(-2)-14$$
$$= -8-3(4)-8-14$$
$$= -8-12-8-14$$
$$= -42$$

Q.2.

(i) If (x+2) is a factor of $3x^2-4kx-4k^2$, then find the value(s) of k.

Sol:

Let $P(x) = 3x^2-4kx-4k^2$

As given that x + 2 is a factor of P(x), so

$R = 0$

i.e. $P(-2) = 0$

So $3(-2)^2-4k(-2)-4k^2 = 0$

$12+8k-4k^2 = 0$

Dividing by 4

$3+2k-k^2 = 0$

$3+3k-k-k^2 = 0$

$3(1+k)-k(1+k) = 0$

$(1+k)(3-k) = 0$

$\Rightarrow 1+k = 0 \text{ or } 3-k = 0$

$\Rightarrow k = -1 \text{ or } k = 3$

(ii) If (x – 1) is factor of $x^3-kx^2+11x-6$ then find the value of k.

Sol:

$P(x) = x^3-kx^2+11x-6$

As given that x – 1 is a factor of P(x), so

$R = 0$

$P(1) = 0$

$(1)^3-k(1)^2+11(1)-6 = 0$

$1-k+11-6 = 0$

$6-k = 0$

$\Rightarrow k = 6$

Q.3 Without actual long division determine whether

(i) (x – 2) and (x – 3) are factors of P $P(x) = x^3-12x^2+44x-48$

Sol:

$P(x)=x^3-12x^2+44x-48$

Taking $x-2$

$R=P(2)$

$=(2)^3-12(2)^2+44(2)-48$

$=8-12(4)+88-48$

$=8-48+88-48$

$=0$

As the remainder is zero, so (x – 2) is a factor of P(x)

Now $P(x)=x^3-12x^2+44x-48$

Taking $x-3$

$R=P(3)$

$=(3)^3-12(3)^2+44(3)-48$

$=27-12(9)+132-48$

$=27-108+132-148$

$=3\neq 0$

As the remainder is not equal to zero, so (x–3) is not a factor of P(x).

(ii) (x – 2), (x + 3) and (x – 4) are factors of $q(x)=x^3+2x^2-5x-6$

Sol:

$q(x)=x^3+2x^2-5x-6$

Taking $x-2$

$R=q(2)=(2)^3+2(2)^2-5(2)-6$

$=8+2(4)-10-6$

$R=0$

As the remainder is zero so $(x-2)$ is a factor of P(x)

Now $q(x)=x^3+2x^2-5x-6$

Taking $x+3$

$R=q(-3)$

$=(-3)^3+2(-3)^2-5(-3)-6$

$=-27+2(9)+15-6$

$=-27+18+15-6$

$=0$

As the remainder is zero, so (x + 3) is a factor of P(x)

Now $q(x)=x^3+2x^2-5x-6$

Taking $x-4$

$R=q(4)$

$=(4)^3+2(4)^2-5(4)-6$

$=64+2(16)-20-6$

$=64+32-20-6$

$=70\neq 0$

As remainder is not equal to zero, so x – 4 is not a factor of P (x)

Q.4 For what value of m is the polynomial $P(x)=4x^3-7x^2+6x-3m$ exactly divisible by x + 2?

Sol:

$m=?$

$P(x)=4x^3-7x^2+6x-3m$

Taking $x+2$

As $p(x)$ is exactly divisible by $(x+2)$, so

$R=0$

$P(-2)=0$

$4(-2)^3-7(-2)^2+6(-2)-3m=0$

$4(-8)-7(4)-12-3m=0$

$-32-28-12-3m=0$

$-72-3m=0$

$-3m=+72$

$$m=\frac{72}{-3}$$

$$m=-24$$

Q.5 Determine the value of k if $P(x)=kx^3+4x^2+3x-4$ and $q(x)=x^3-4x+k$. Leaves the same remainder when divided by x – 3.

Sol:

K = ?

When p(x) is divided by (x–3) by remainder theorem then remainder is

$$R_1=P(3)$$

$$=k(3)^3+4(3)^2+3(3)-4$$

$$=27k+36+9-4$$

$$=27k+41$$

When q(x) is divided by (x–3) by remainder theorem then remainder is

$$R_2=q(3)$$

$$q(x)=x^3-4x+k$$

$$=(3)^3-4(3)+k$$

$$=27-12+k$$

$$=15+k$$

As given that when P(x) and q(x) are divided by x – 3, then remainder is same, so

$$R_1 = R_2$$

$$27k+41=15+k$$

$$27k-k=15-41$$

$$26k=-26$$

$$k=\frac{-26}{26}$$

$$\boxed{k=-1}$$

Q.6

The remainder of dividing the polynomial $P(x)=x^3+ax^2+7$ by (x + 1) is 2b. calculate the value of 'a' and 'b' if this expression leaves a remainder of (b + 5) on being divided by (x – 2)

Sol:

$$P(x)=x^3+ax^2+7$$

The remainder by dividing $P(x)$ by $x+1$ is $2b$, so

$$P(-1)=2b$$

$$(-1)^3+a(-1)^2+7=2b$$

$$-1+a+7=2b$$

$$a+6=2b$$

$$a-2b=-6 \text{......(i)}$$

Taking $x-2$

The remainder by dividing $P(x)$ by $(x-2)$ is $(b+5)$, so

$$P(2)=b+5$$

$$(2)^3+a(2)^2+7=b+5$$

$$8+4a+7=b+5$$

$$4a+15=b+5$$

$$4a-b=5-15$$

$$4a-b=-10 \text{ .....(ii)}$$

Multiplying (ii) by 2

$$8a-2b=-20 \text{ ..... (iii)}$$

By Subtracting , (iii) from (i)

$$\begin{array}{rrl} a & -2b & = -6 \\ 8a & \mp 2b & = \mp 20 \\ \hline -7a & & = 14 \end{array}$$

$$a=-\frac{14}{7}=-2$$

Putting (1)

$a-2b=-6$
$-2-2b=-6$
$-2b=-6+2$
$-2b=-4$
$b=2$

Q.7 The polynomial $x^3+\ell x^2+mx+24$ has a factor (x + 4) and it leaves a remainder of 36 when divided by (x–2). Find the value of ℓ and m.

Sol:

Let $P(x)=x^3+\ell x^2+mx+24$

As $(x+4)$ is a factor of P(x),

So remainder will be zero.i.e

$R=P(-4)=0$

$P(-4)=0$

$(-4)^3+\ell(-4)^2+m(-4)+24=0$

$-64+16\ell-4m+24=0$

$16\ell-4m-40=0$

$16\ell-4m=40$

Dividing by 4

$4\ell-m=10$.....(i)

Now as given that P(x) is divided by (x – 2) leaves a remainder 36, so

$R=36$

i.e. $P(2)=36$

$(2)^3+\ell(2)^2+m(2)+24=36$

$8+4\ell+2m+24=36$

$4\ell+2m+32=36$

$4\ell+2m=36-32$

$4\ell+2m=4$

Dividing by 2

$2\ell+m=2$.......(ii)

Adding (i) and (ii)

$$\begin{array}{l} 4\ell-m=10 \\ \underline{2\ell+m=2} \\ 6\ell \quad =12 \end{array}$$

$\ell=\dfrac{12}{6}$

$\ell=2$

Putting value of 'ℓ' in (ii)

$2\ell+m=2$

$2(2)+m=2$

$m=2-4$

$m=-2$

Q.8. The Expression ℓx^3+mx^2-4 leaves remainder of –3 and 12 when divided by (x–1) and (x+2) respectively. Calculate the values of ℓ and m.

Sol:

Let $P(x)=\ell x^3+mx^2-4$

As given that P(x) when divided by x – 1 leaves remainder –3, so

$R=-3$

$P(1)=-3$

$\ell(1)^3+m(1)^2-4=-3$

$\ell+m-4=-3$

$\ell+m=4-3$

$\ell+m=1$...........(i)

As given that P(x) when divided by (x + 2) leaves the remainder 12, so

$R=12$

$P(-2)=12$

$\ell(-2)^3+m(-2)^2-4=12$

$-8\ell+4m-4=12$

$-8\ell+4m=12+4$

$-8\ell+4m=16$

Dividing by 4

$-2\ell+m=4$........(ii)

Subtracting (ii) from (i)

$$\begin{array}{l} \ell+m=1 \\ -2\ell+m=4 \\ \underline{+\quad -\quad -} \\ 3\ell \quad =-3 \end{array}$$

$$\ell=\frac{-3}{3}$$

$$\ell=-1$$

Putting value of 'ℓ' in (i)

$\ell+m=1$

$-1+m=1$

$m=1+1$

$m=2$

Q.9 The expression $ax^3-9x^2+bx+3a$ is exactly divisible by x^2-5x+6. Find the values of a and b

Sol:

Let $P(x)=ax^3-9x^2+bx+3a$

Taking x^2-5x+6

$=x^2-2x-3x+6$

$=x(x-2)-3(x-2)$

$=(x-2)(x-3)$

As given that P(x) is exactly divisible by (x – 2), so $P(2)=0$

$a(2)^3-9(2)^2+b(2)+3a=0$

$8a-36+2b+3a=0$

$11a+2b=36$.........(i)

As given that P(x) is exactly divisible by x–3, so

$P(3)=0$

$a(3)^3-9(3)^2+b(3)+3a=0$

$27a-81+3b+3a=0$

$30a+3b=81$

Dividing by 3

$10a+b=27$......(ii)

Multiplying (ii) by 2 and subtracting (i) from it.

$$\begin{array}{l} 20a+2b=54 \\ 11a+2b=36 \\ \underline{-\quad -\quad -} \\ 9a \quad =18 \end{array}$$

$$a=\frac{18}{9}$$

$$a=2$$

Putting value of 'a' in (ii)

$10a+b=27$

$10(2)+b=27$

$b=27-20$

$b=7$

Rational Root Theorem

Let

$a_0x^n+a_1x^{n-1}+\dots\dots+a_{n-1}x+a_n=0,\quad a_0\neq 0$

be a polynomial equation of degree n with integral coefficients. If p/q is a rational root (expressed in lowest terms) of the equation, then p is a factor of the constant term a_n and q is a factor of the leading coefficient a_0.

Example

Factorize the polynomial x^3-4x^2+x+6, by using Factor Theorem.

Solution:

We have $P(x)=x^3-4x^2+x+6$.

Possible factors of the constant term p = 6 are ±1, ±2, ±3, and ±6 and of leading coefficient q = 1 are ±1. Thus the expected zeros (or roots) of $P(x)=0$ are $\frac{p}{q}=\pm1,\pm2,\pm3$ and ±6. If $x=a$ is a zero of $P(x)$, then $(x-a)$ will be a factor.

We use the hit and trial method to find zeros of P(x). Let us try x = 1.

Now $P(1)=(1)^3-4(1)^2+1+6$

$=1-4+1+6$

$=4\neq0$

Hence x = 1 is not a zero of P(x).

Again $P(-1)=(-1)^3-4(-1)^2-1+6$

$=-1-4-1+6=0$

Hence $x=-1$ is a zero of P(x) and therefore,

$x-(-1)=(x+1)$ is a factor of P(x).

Now $P(2)=(2)^3-4(2)^2+2+6$

$=8-16+2+6=0 \Rightarrow x=2$ is a root.

Hence $(x-2)$ is also a factor of P(x).

Similarly $P(3)=(3)^3-4(3)^2+3+6$

$=27-36+3+6=0 \Rightarrow x=3$ is a zero of P(x).

Hence $(x-3)$ is the third factor of P(x).

Thus the factorized form of

$P(x)=x^3-4x^2+x+6$ *is* $(x+1)(x-2)(x-3)$.

Exercise 5.4

Factorize each of the following cubic polynomials by factor theorem.

Q.1 x^3-2x^2-x+2

Let $P(x)=x^3-2x^2-x+2$

Put $x=1$

$P(1)=(1)^3-2(1)^2-(1)+2$

$=1-2-1+2$

$=-3+3=0$

As, R = 0,

So $(x-1)$ is a factor

Put $x=-1$

$P(-1)=(-1)^3-2(-1)^2-(-1)+2$

$=-1-2+1+2$

As R = 0,

So (x+1) is the second factor of p(x).

Put $x=2$

$P(2)=(2)^3-2(2)^2-(2)+2$

$=8-8-2+2$

$=10-10$

$=0$

As R = 0,

So $(x-2)$ is the third factor

Hence $P(x)=x^3-2x^2-x+2$

$=(x-1)(x+1)(x-2)$

Q.2 $x^3-x^2-22x+40$

Sol:

Let $P(x)=x^3-x^2-22x+40$

Put $x=1$

$P(1)=(1)^3-(1)^2-22(1)+40$

$=1-1-22+40$

$=18\neq0$

Hence $x-1$ is not a zero of $P(x)$

Put $x=-1$

$P(-1)=(-1)^3-(-1)^2-22(-1)+40$

$=-1-1+22+40$

$=60\neq0$

Hence $x=-1$ is not a zero of $P(x)$

Put $x=2$

$P(2)=(2)^3-(2)^2-22(2)+40$

$=8-4-44+40=0$

Hence $x-2$ is a zero of $P(x)$

So $(x-2)$ is a factor

Put $x=-2$

$P(-2)=(-2)^3-(-2)^2-22(-2)+40$

$=-8-4+44+40=72$

Hence $x=-2$ is not a zero of $P(x)$

Put $x=3$

$P(3)=(3)^3-(3)^2-22(3)+40$

$=27-9-66+40$

$=67-75$

$=-8\neq0$

Hence $x=3$ is not a zero of $P(x)$

Put $x=-3$

$P(-3)=(-3)^3-(-3)^2-22(-3)+40$

$=-27-9+66+40$

$=106-36$

$=70\neq0$

Hence $x=-3$ is not a zero of $P(x)$

Put $x=4$

$P(4)=(4)^3-(4)^2-22(4)+40$

$=64-16-88+40$

$=104-104$

$=0$

Hence $x=4$ is a zero of $P(x)$

So $(x-4)$ is second factor

Put $x=-4$

$P(-4)=(-4)^3-(-4)^2-22(-4)+40$

$=-64-16+88+40$

$=-80+128$

$=48\neq0$

So, $x=-4$ is not a zero of $P(x)$

Put $x=5$

$P(5)=(5)^3-(5)^2-22(5)+40$

$=125-25-110+40$

$=165-135$

$=30\neq0$

So, $x=5$ is not a zero of $P(x)$

Put $x=-5$

$P(-5)=(-5)^3-(5)^2-22(-5)+40$

$=-125-25+110+40$

$=-150+150$

$=0$

So, $x=-5$ is a zero of $P(x)$

Hence $x+5$ is third factor of $P(x)$

Hence $P(x)=x^3-x^2-22x+40$

$=(x-2)(x-4)(x+5)$

Q.3 $x^3-6x^2+3x+10$

Sol:

Let $P(x)=x^3-6x^2-6x^2+3x+10$

Put $x=1$

$P(1)=(1)^3-6(1)^2+3(1)+10$

$=1-6+3+10$

$=14-6$
$=8\neq 0$

So, $x=1$ is not a zero of $P(x)$

Put $x=-1$

$P(-1)=(-1)^3-6(-1)^2+3(-1)+10$
$=-1-6-3+10$
$=-10+10$
$=0$

So, $x=-1$ is a zero of $P(x)$.

Hence $(x+1)$ is a factor of $P(x)$

Put $x=2$

$P(2)=(2)^3-6(2)^2+3(2)+10$
$=8-24+6+10$
$=24-24$
$=0$

So, $x=2$ is a zero of $P(x)$.

Hence $(x-2)$ is second factor of $P(x)$

Put $x=-2$

$P(-2)=(-2)^3-6(-2)^2+3(-2)+10$
$=-8-24-6+10$
$=-28\neq 0$

So, $x=-2$ is not a zero of $P(x)$

Put $x=3$

$P(3)=(3)^3-6(3)^2+3(3)+10$
$=27-6(9)+9+10$
$=46-54$
$=-8\neq 0$

So, $x=3$ is not a zero of $P(x)$

Put $x=-3$

$P(-3)=(-3)^3-6(-3)^2+3(-3)+10$
$=-27-6(9)-9+10$
$=-90+10$
$=-80\neq 0$

So, $x=-3$ is not a zero of $P(x)$

Put $x=4$

$P(4)=(4)^3-6(4)^2+3(4)+10$
$=64-6(16)+12+10$
$=86-96$
$=-10\neq 0$

So, $x=4$ is not a zero of $P(x)$

Put $x=-4$

$P(-4)=(-4)^3-6(-4)^2+3(-4)+10$
$=-64-6(16)-12+10$
$=-64-96-12+10$
$=-172+10$
$=-162$
$=-162\neq 0$

Put $x=5$

$P(5)=(5)^3-6(5)^2+3(5)+10$
$=125-150+15+10$
$=150-150$
$=0$

So, $x=5$ is a zero of $P(x)$

Hence $(x-5)$ is third factor of $P(x)$

Hence $P(x)=x^3-6x^2+3x+10$
$=(x+1)(x-2)(x-5)$

Q.4 $x^3+x^2-10x+8$

Sol:

Let $P(x)=x^3+x^2-10x+8$

Put $x=1$

$P(1)=(1)^3+(1)^2-10(1)+8$
$=1+1-10+8$
$=0$

So, $x=1$ is a zero of $P(x)$

Hence $(x-1)$ is a factor of $P(x)$

Put $x=-1$

$$P(-1)=(-1)^3+(-1)^2-10(-1)+8$$
$$=-1+1+10+8$$
$$=18\neq 0$$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$$P(2)=(2)^3+(2)^2-10(2)+8$$
$$=8+4-20+8$$
$$=20-20$$
$$=0$$

So, $x=2$ is a zero of $P(x)$

Hence $x-2$ is second factor of $P(x)$

Put $x=-2$

$$P(-2)=(-2)^3+(-2)^2-10(-2)+8$$
$$=-8+4+20+8$$
$$=24\neq 0$$

So, $x=-2$ is not a zero of $P(x)$

Put $x=3$

$$P(3)=(3)^3+(3)^2-10(3)+8$$
$$=27+9-30+8$$
$$=44-30$$
$$=14\neq 0$$

Put $x=-3$

$$P(-3)=(-3)^3+(-3)^2-10(-3)+8$$
$$=-27+9+30+8$$
$$=-27+47$$
$$=20\neq 0$$

So, $x=-3$ is not a zero of $P(x)$

Put $x=4$

$$P(4)=(4)^3+(4)^2-10(4)+8$$
$$=64+16-40+8$$
$$=88-40$$
$$=48\neq 0$$

So, $x=4$ is not a zero of $P(x)$

Put $x=-4$

$$P(-4)=(-4)^3+(-4)^2-10(-4)+8$$
$$=-64+16+40+8$$
$$=-64+64$$
$$=0$$

So, $x=-4$ is a zero of $P(x)$

Hence $x+4$ is third factor of $P(x)$

Hence $P(x)=x^3+x^2-10x+8$
$$=(x-1)(x-2)(x+4)$$

Q.5 x^3-2x^2-5x+6

Sol:

$P(x)=x^3-2x^2-5x+6$

Put $x=1$

$$P(1)=(1)^3-2(1)^2-5(1)+6$$
$$=1-2-5+6$$
$$=7-7$$
$$=0$$

So, $x=1$ is a zero of $P(1)$

Hence $x-1$ is a factor of $P(x)$

Put $x=-1$

$$P(-1)=(-1)^3-2(-1)^2-5(-1)+6$$
$$=-1-2+5+6$$
$$=-3+11$$
$$=8\neq 0$$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$$P(2)=(2)^3-2(2)^2-5(2)+6$$

$=8-8-10+6$

$=-4\neq 0$

So, $x=2$ is not a zero of $P(x)$

Put $x=-2$

$P(-2)=(-2)^3-2(-2)^2-5(-2)$

$=-8-8+10+6$

$=0$

So, $x=-2$ is a zero of $P(x)$

Hence $(x+2)$ is second factor of $P(x)$

Put $x=3$

$P(3)=(3)^3-2(3)^2-5(3)+6$

$=27-18-15+6$

$=33-33$

$=0$

So, $x=3$ is a zero of $P(x)$

Hence $(x-3)$ is third factor of $P(x)$

Hence $P(x)=x^3-2x^2-5x+6$

$=(x-1)(x+2)(x-3)$

Q.6 $x^3+5x^2-2x-24$

Sol:

Let $P(x)=x^3+5x^2-2x-24$

Put $x=1$

$P(1)=(1)^3+5(1)^2-2(1)-24$

$=1+5-2-24$

$=6-26$

$=-20\neq 0$

So, $x=1$ is not a zero of $P(x)$

Put $x=-1$

$P(-1)=(-1)^3+5(-1)^2-2(-1)-24$

$=-1+5+2-24$

$=7-25$

$=-18\neq 0$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$P(2)=(2)^3+5(2)^2-2(2)-24$

$=8+20-4-24$

$=28-28$

$=0$

So, $x=2$ is a zero of $P(x)$

Hence $(x-2)$ is a factor of $P(x)$

Put $x=-2$

$P(-2)=(-2)^3+5(-2)^2-2(-2)-24$

$=-8+5(4)+4-24$

$=-32+24$

$=-8\neq 0$

So, $x=-2$ is not a zero of $P(x)$

Put $x=3$

$P(3)=(3)^3+5(3)^2-2(3)-24$

$=27+5(9)-6-24$

$=72-30$

$=42\neq 0$

So, $x=3$ is not a zero of $P(x)$

Put $x=-3$

$P(-3)=(-3)^3+5(-3)^2-2(-3)-24$

$=-27+45+6-24$

$=51-51$

$=0$

So, $x=-3$ is a zero of $P(x)$

Hence $(x+3)$ is second factor of $P(x)$

Put $x=4$

$P(4)=(4)^3+5(4)^2-2(4)-24$

$=64+5(16)-8-24$

$=144-32$

$=112\neq 0$

So, $x=4$ is not a zero of $P(x)$

Put $x=-4$

$P(-4)=(-4)^3+5(-4)^2-2(-4)-24$

$=-64+80+8-24$

$=0$

So, $x=-4$ is a zero of $P(x)$

Hence $(x+4)$ is third factor of $P(x)$

Hence $P(x)=x^3+5x^2-2x-24$

$=(x-2)(x+3)(x+4)$

Q.7 $3x^3-x^2-12x+4$

Sol: $P(x)=3x^3-x^2-12x+4$

Put $x=1$

$P(1)=3(1)^3-(1)^2-12(1)+4$

$=3-1-12+4$

$=7-13$

$=-6\neq 0$

So, $x=1$ is not a zero of $P(x)$

Put $x=-1$

$P(-1)=3(-1)^3-(-1)^2-12(-1)+4$

$=-3-1+12+4$

$=-4+16$

$=12\neq 0$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$P(2)=3(2)^3-(2)^2-12(2)+4$

$=24-4-24+4$

$=28-28$

$=0$

So, $x=2$ is a zero of $P(x)$

Hence $(x-2)$ is a factor of $P(x)$

Put $x=-2$

$P(-2)=3(-2)^3-(-2)^2-12(-2)+4$

$=-24-4+24+4$

$=-28+28$

$=0$

So, $x=-2$ is a zero of $P(x)$

Hence $(x+2)$ is second factor of $P(x)$

Put $3x=1$

$x=\frac{1}{3}$

$P\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)^3-\left(\frac{1}{3}\right)^2-12\left(\frac{1}{3}\right)+4$

$=\cancel{3}\left(\frac{1}{27}\right)-\frac{1}{9}-12\left(\frac{1}{3}\right)+4$

$=\frac{1}{9}-\frac{1}{9}-4+4$

$=0$

So, $x=\frac{1}{3}$ is a zero of $P(x)$

Hence $(3x-1)$ is third factor of $P(x)$

Hence $P(x)=3x^3-x^2-12x+4$

$=(x-2)(x+2)(3x-1)$

Q.8 $2x^3+x^2-2x-1$

Let $P(x)=2x^3+x^2-2x-1$

Put $x=1$

$P(1)=2(1)^3+(1)^2-2(1)-1$

$=2+1-2-1$

$=3-3$

$=0$

So, $x=1$ is a zero of $P(x)$

Hence $(x-1)$ is a factor of $P(x)$

Put $x=-1$

$$P(-1)=2(-1)^3+(-1)^2-2(-1)-1$$
$$=-2+1+2-1$$
$$=-1+1$$
$$=0$$

So, $x=-1$ is a zero of $P(x)$

Hence $(x+1)$ is second factor of $P(x)$

Put $2x=1$

$$x=\frac{1}{2}$$

$$P\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)-1$$
$$=2\left(\frac{1}{8}\right)+\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)-1$$
$$=\frac{1}{4}+\frac{1}{4}-1-1$$
$$=\frac{-3}{2}\neq 0$$

So, $x-2$ is not a zero of $P(x)$

Put $x=\frac{-1}{2}$

$$P\left(\frac{-1}{2}\right)=2\left(\frac{-1}{2}\right)^3+\left(\frac{-1}{2}\right)^2-2\left(\frac{-1}{2}\right)-1$$
$$=2\left(-\frac{1}{8}\right)+\frac{1}{4}+1-1$$
$$=-\frac{1}{4}+\frac{1}{4}+1-1$$
$$=0$$

So, $x=\frac{-1}{2}$ is a zero of $P(x)$

Hence $2x+1$ is third factor of $P(x)$

Hence $P(x)=2x^3+x^2-2x-1$

$$=(x-1)(x+1)(2x+1)$$

Objective

1. The factor of x^2-5x+6 are: ____

(a) $x+1, x-6$ (b) $x-2, x-3$

(c) $x+6, x-1$ (d) $x+2, x+3$

2. Factors of $8x^3+27y^3$ are:____

(a) $(2x+3y)(4x^2-9y^2)$

(b) $(2x-3y)(4x^2-9y^2)$

(c) $(2x+3y)(4x^2-6xy+9y^2)$

(d) $(2x-3y)(4x^2+6xy+9y^2)$

3. Factors of $3x^2-x-2$ are:

(a) $(x+1)(3x-2)$ (b) $(x+1)(3x+2)$

(c) $(x-1)(3x-2)$ (d) $(x-1)(3x+2)$

4. Factors of a^4-4b^4 are: ____

(a) $(a-b)(a+b)(a^2+4b^2)$

(b) $(a^2-2b^2)(a^2+2b^2)$

(c) $(a-b)(a+b)(a^2-4b^2)$

(d) $(a-2b)(a^2+2b^2)$

5. What will be added to complete the square of $9a^2-12ab$?____

(a) $-16b^2$ (b) $16b^2$

(c) $4b^2$ (d) $-4b^2$

6. Find m so that x^2+4x+m is a complete square:

(a) 8 (b) −8

(c) 4 (d) 16

7. Factors of $5x^2-17xy-12y^2$ are____

(a) $(x+4y)(5x+3y)$

(b) $(x-4y)(5x-3y)$

(c) $(x-4y)(5x+3y)$

(d) $(5x-4y)(x+3y)$

8. Factors of $27x^3 - \frac{1}{x^3}$ are___

(a) $\left(3x - \frac{1}{x}\right)\left(9x^2 + 3 + \frac{1}{x^2}\right)$

(b) $\left(3x + \frac{1}{x}\right)\left(9x^2 + 3 + \frac{1}{x^2}\right)$

(c) $\left(3x - \frac{1}{x}\right)\left(9x^2 - 3 + \frac{1}{x^2}\right)$

(d) $\left(3x + \frac{1}{x}\right)\left(9x^2 - 3 + \frac{1}{x^2}\right)$

9. If $x - 2$ is a factor of $p(x) = x^2+2kx+8$, then K = ___

(a) -3 (b) 3
(c) 4 (d) 5

10. $4a^2+4ab+(\ldots..)$ is a complete square

(a) b^2 (b) 2b
(c) a^2 (d) $4b^2$

11. $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2} = \ldots\ldots\ldots$

(a) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$ (b) $\left(\frac{x}{y} + \frac{y}{x}\right)^2$

(c) $\left(\frac{x}{y} - \frac{y}{x}\right)^3$ (d) $\left(\frac{x}{y} + \frac{y}{x}\right)^3$

12. $(x+y)(x^2 - xy + y^2) =$ _____

(a) $x^3 - y^3$ (b) $x^3 + y^3$
(c) $(x+y)^3$ (d) $(x-y)^3$

13. Factors of $x^4 - 16$ is ____

(a) $(x-2)^2$
(b) $(x-2)(x+2)(x^2+4)$
(c) $(x-2)(x+2)$
(d) $(x+2)^2$

14. Factors of $3x - 3a + xy - ay$.

(a) $(3+y)(x-a)$
(b) $(3-y)(x+a)$
(c) $(3-y)(x-a)$
(d) $(3+y)(x+a)$

15. Factors of $pqr + qr^2 - pr^2 - r^3$ is:

(a) $r(p+r)(q-r)$ (b) $r(p-r)(q+r)$
(c) $r(p-r)(q-r)$ (d) $r(p+r)(q+r)$

Answer Key

1.	b	**2.**	c	**3.**	d	**4.**	b	**5.**	c
6.	c	**7.**	c	**8.**	a	**9.**	a	**10.**	a
11.	a	**12.**	b	**13.**	b	**14.**	a	**15.**	a